4 $f(-x) = \frac{-x}{-x-1} = \frac{x}{x+1}$ geen symmetrie, geen periode

5 $\lim\limits_{x\to\infty} \frac{x}{x-1} = \lim\limits_{x\to\infty} \frac{\frac{x}{x}}{\frac{x}{x}-\frac{1}{x}} = \frac{1}{1} = 1$

$\lim\limits_{x\to-\infty} \frac{x}{x-1} = \lim\limits_{x\to-\infty} \frac{1}{1} = 1$

horizontale asymptoot $y=1$.

6 $f'(x) = \frac{(x-1)\cdot 1 - x\cdot 1}{(x-1)^2} = \frac{-1}{(x-1)^2}$

teller — — — — — — — —

noemer + + + + 0 + + + +

$f'(x)$ — — — — x — — — —

geen max/min

7 $f''(x) = \left[-(x-1)^{-2}\right]' = 2(x-1)^{-3}$

teller + + + + + + + + +

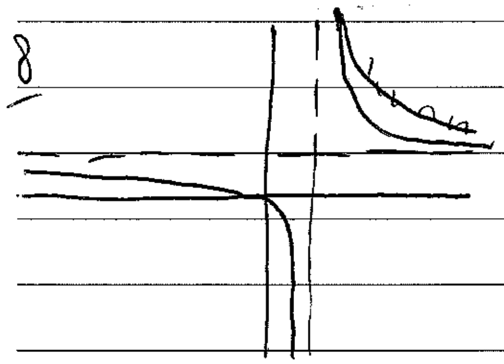
noemer — — — — | + + + + + +

$f''(x)$ — — — — — x + + + + +

concaaf convex

geen buigpunt want $f(1)$ bestaat niet.

8



Bereik

$\{y \in \mathbb{R} \mid y \neq 1\}$

DE HAAGSE HOGESCHOOL

Tentamenpapier

Naam		Datum	5-4-2012
Opleiding	Mechatronica	Vak (code)	MEWIS3
Id-code		Tentamennr.	T1-1 Cijfer
Klas	MeP2	Afdeling	
Docent	Smit	Module	

Vraag 1

a) $(2+i)(6+3i) = 12+6i+6i+3i^2 =$
$12+12i-3 = 9+12i$

b) $\frac{5+3i}{2+i} = \frac{5+3i}{2+i}\cdot\frac{2-i}{2-i} = \frac{13+i}{5} = \frac{13}{5}+\frac{1}{5}i$

c) $(2+2i)^8$ $\quad |2+2i| = \sqrt{2^2+2^2} = \sqrt{8}$
$\arg(2+2i) = \arctan\frac{2}{2} = \frac{\pi}{4}$

$(2+2i)^8 = \left(\sqrt{8}\,e^{i\pi/4}\right)^8 = \left(\sqrt{8}\right)^8 \cdot e^{i 8 \frac{\pi}{4}} = 8^4 =$
4096

d) $z^3 = 8 = 8\cdot e^{i(0+2k\pi)}$
$z = 2e^{i\frac{2}{3}k\pi} =$
$z = 2 \lor z = 2e^{i\frac{2}{3}\pi} \lor z = 2e^{i\frac{4}{3}\pi}$
$z = 2 \lor z = -1+\sqrt{3}i \lor z = -1-\sqrt{3}i$

Vraag 2

$f(t) = \tan(2t)$

$f'(t) = \frac{1}{\cos^2 2t} \cdot 2$

$f''(t) = \left[2\cos^{-2}(2t)\right]' =$

$-2 \cdot 2\cos^{-3}(2t) \cdot -\sin(2t) \cdot 2 =$

$\frac{8\sin(2t)}{\cos^3(2t)}$

$f'''(t) = \left(\frac{naT - taN}{N^2}\right) = \frac{\cos^3(2t) \cdot 8\cos(2t) \cdot 2 - 8\sin 2t \cdot 3\cos^2(2t) \cdot -\sin 2t \cdot 2}{\cos^6 2t}$

$= \frac{16\cos^4(2t) + 48\sin^2(2t)\cos^2(2t)}{\cos^6(2t)}$

$= \frac{16}{\cos^2(2t)} + \frac{48\sin^2(2t)}{\cos^4(2t)}$

Vraag 3

a) $\int_1^2 \frac{6x^2+4x}{x^3+x^2+8}\,dx =$ $\quad p = x^3+x^2+8$, $\quad dp = (3x^2+2x)\,dx$

grenzen $1^3+1^2+8 = 10$ $\quad 2^3+2^2+8 = 20$.

$\int_{p=10}^{20} \frac{6x^2+4x}{p} \cdot \frac{dp}{3x^2+2x} = 2\int_{10}^{20} \frac{1}{p}\,dp = 2\left[\ln|p|\right]_{10}^{20}$

$= 2(\ln 20 - \ln 10) = 2\ln\frac{20}{10} = 2\ln 2.$

Vraag 3

b) $\int z^7 \ln z\,dz = \int \ln z\; d\left(\frac{1}{8}z^8\right) =$

$\frac{1}{8}z^8 \ln z - \int \frac{1}{8}z^8\, d(\ln z) =$

$\frac{1}{8}z^8 \ln z - \frac{1}{8}\int z^7\,dz =$

$\frac{1}{8}z^8 \ln z - \frac{1}{64}z^8 + C$

c) $\int \frac{1}{t\ln^2 t}\,dt =$ $\quad p = \ln t$, $\quad dp = \frac{1}{t}\,dt$, $\quad (dt = t\,dp)$

$\int \frac{1}{t\,p^2}\, t\,dp = \int \frac{1}{p^2}\,dp = \int p^{-2}\,dp$

$= -p^{-1} + C = -\frac{1}{\ln t} + C$

~~4~~ Vraag 4

1. Voor domein $x-1 \neq 0$, $x \neq 1$ $\quad \{x \in \mathbb{R} \mid x \neq 1\}$

$\lim_{x \downarrow 1} \frac{x}{x-1} = \left(\frac{1}{0^+}\right) = +\infty$

$\lim_{x \uparrow 1} \frac{x}{x-1} = \left(\frac{1}{0^-}\right) = -\infty$

verticale asymptoot op $x = 1$

2. $x = 0 \Rightarrow y = 0$ $\quad (0,0)$ snijpunt y-as

$y = 0 \Rightarrow x = 0$ $\quad (0,0)$ snijpunt x-as

3. teller $\quad$ − − − 0 + + +

noemer $\quad$ − − − − 0 + + +

$f(x)$ $\quad$ + + + + 0 − x + + + +

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Vraag 4

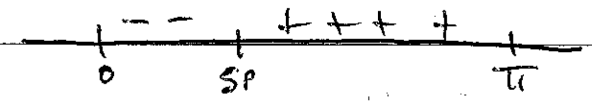
b) 1 domein gegeven $[0, \pi]$
geen verticale asymptoten.

2 snijpunt y-as $g(0) = 0 + \sin \pi = 0$ $(0,0)$

snijpunt x-as $x + \sin(2x + \pi) = 0$

met GR $(0,0)$ en $(0{,}94;0)$

3



4 geen symmetrie t.o.v. assen of O
~~5~~ geen periode
5 geen hor asymptoten (beperkt domein)
6 $f'(x) = 1 + 2\cos(2x + \pi)$

$$1 + 2\cos(2x + \pi) = 0$$
$$\cos(2x + \pi) = -\tfrac{1}{2}$$
$$\cos(2x + \pi) = \cos\left(\tfrac{2}{3}\pi\right)$$
$$2x + \pi = \tfrac{2}{3}\pi + k \cdot 2\pi \quad \vee \quad 2x + \pi = -\tfrac{2}{3}\pi + k \cdot 2\pi$$
$$x = -\tfrac{1}{6} + k\pi \quad \vee \quad x = -\tfrac{5}{6}\pi + k\pi$$

dus in ~~twee~~ domein

$x = \frac{1}{6}\pi \quad \vee \quad x = \frac{5}{6}\pi$.

$f'(x)$: $-\,-\,-$ on $(0, \frac{1}{6}\pi)$, $+\,+\,+\,+\,+\,+$ on $(\frac{1}{6}\pi, \frac{5}{6}\pi)$, $-\,-\,-$ on $(\frac{5}{6}\pi, \pi)$

$0 \quad \frac{1}{6}\pi \quad \frac{5}{6}\pi \quad \pi$

maximum globaal

$f(\frac{5}{6}\pi) = \frac{5}{6}\pi + \sin(\frac{10}{6}\pi + \pi) = \frac{5}{6}\pi + \frac{1}{2}\sqrt{3}$

minimum globaal

$f(\frac{1}{6}\pi) = \frac{1}{6}\pi + \sin(\frac{2}{6}\pi + \pi) = \frac{1}{6}\pi - \frac{1}{2}\sqrt{3}$

rand maximum

$f(0) = 0$

rand minimum

$f(\pi) = \pi + \sin(2\pi + \pi) = \pi$

7 $f''(x) = -4\sin(2x+\pi)$

$-4\sin(2x+\pi) = 0$

$\sin(2x+\pi) = \sin 0$

$2x+\pi = 0 + k\cdot 2\pi \quad \vee \quad 2x+\pi = \pi - 0 + k\cdot 2\pi$

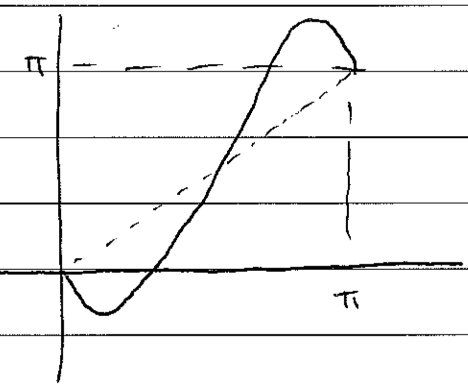
$x = -\frac{1}{2}\pi + k\pi \quad \vee \quad x = 0 + k\pi$

buigpunten $x = 0 \vee x = \frac{1}{2}\pi \vee x = \pi$

$f''(x)$: 0 at 0, $+\,+\,+$ on $(0, \frac{\pi}{2})$, 0 at $\frac{\pi}{2}$, $-\,-\,-$ on $(\frac{\pi}{2}, \pi)$, 0 at π

$0 \quad \pi/2 \quad \pi$

8



9 Bereik

$\{y \in \mathbb{R} \mid \frac{1}{6}\pi - \frac{1}{2}\sqrt{3} < y < \frac{5}{6}\pi + \frac{1}{2}\sqrt{3}\}$